Deflection of Beams

Worked Example 1

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MMME2053 – Mechanics of Solids

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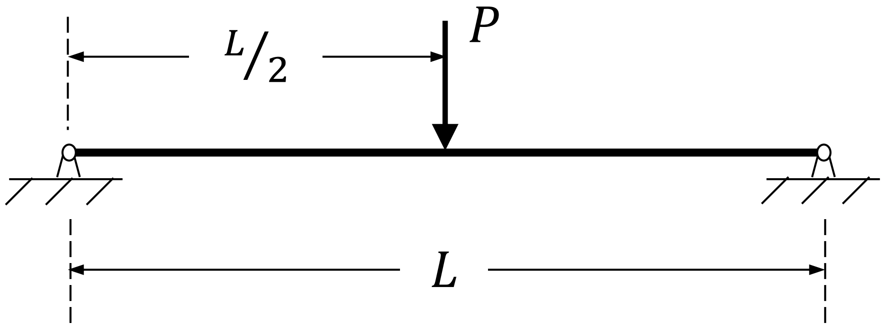
Worked Example 1

Simply Supported, Single Point Loaded Beam

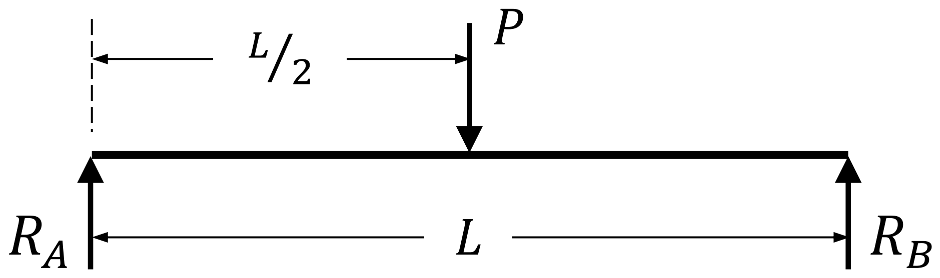
The beam shown below is the example simply supported, centrally point loaded beam used in the description of Macaulay's method in section 3 of the notes.



Problem

Use Macaulay's method to determine expressions for the slope and deflection at the position of the point load.

Drawing a free body diagram of the beam:



Determination of Reaction Forces

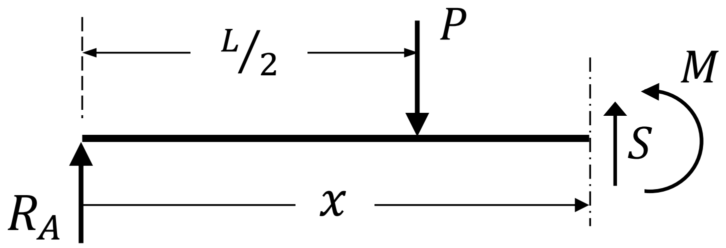
Vertical equilibrium: $P = R_A + R_B \qquad (12)$

Taking moments about the position of R_A: $R_B L = \frac{PL}{2} \qquad \therefore R_B = \frac{P}{2}$

Substituting this into equation (12): $P = R_A + \frac{P}{2} \qquad \therefore R_A = \frac{P}{2}$

Determination of Expression for Bending Moment

Taking the left-hand end of the beam as the origin, sectioning after the final discontinuity and drawing a free body diagram:



Taking moments about the section position in order to determine an expression for the bending moment, M:

$$M = R_A x - P\left\langle x - \frac{L}{2}\right\rangle$$

Determination of 2nd order differential expression for the shape of the beam

Substituting this into the 2nd order differential equation of the elastic line:

$$EI\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = R_A x - P\left\langle x - \frac{L}{2}\right\rangle$$

Integrating to Obtain Expressions for Slope and Deflection

Integrating with respect to x:

$$EI\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{R_A x^2}{2}-\frac{P\left\langle x-\frac{L}{2}\right\rangle^2}{2}+\mathrm{A} \qquad (14)$$

Integrating with respect to x again:

$$EIy=\frac{R_A x^3}{6}-\frac{P\left\langle x-\frac{L}{2}\right\rangle^3}{6}+\mathrm{A}x+\mathrm{B} \qquad (15)$$

Use of Boundary Conditions to Solve for Constants of Integration

As the beam is simply supported at each end, the boundary conditions are:

BC1: at x = 0, y = 0 applying this into equation (15): $EI\times 0=\frac{R_A\times 0^3}{6}-\frac{P\left\langle 0-\frac{L}{2}\right\rangle^3}{6}+\mathrm{A}\times 0+\mathrm{B}$ $\therefore \mathrm{B}=0$

BC2: at x = L, y = 0 applying this into equation (15): $EI\times 0=\frac{R_A\times L^3}{6}-\frac{P\left(\frac{L}{2}\right)^3}{6}+\mathrm{A}L+0$ $\therefore \mathrm{A}=\frac{L^2}{6}\left(\frac{P}{8}-R_A\right)$

Evaluation of Slope and Deflection at $x = \frac{L}{2}$

Applying $x = \frac{L}{2}$ into equations (14) and (15) gives:

$$EI\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{R_A\left(\frac{L}{2}\right)^2}{2} - \frac{P\,\langle\frac{L}{2}-\frac{L}{2}\rangle^2}{2} + \mathrm{A} \qquad \therefore EI\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{R_A L^2}{8} + \mathrm{A}$$

and

$$EIy = \frac{R_A\left(\frac{L}{2}\right)^3}{6} - \frac{P\,\langle\frac{L}{2}-\frac{L}{2}\rangle^3}{6} + \frac{\mathrm{A}L}{2} \qquad \therefore EIy = \frac{R_A L^3}{48} + \frac{\mathrm{A}L}{2}$$

Substituting the expressions for R_A and A into these, and rearranging for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and y, gives:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{EI}\left(\frac{\left(\frac{P}{2}\right)L^2}{8} + \frac{L^2}{6}\left(\frac{P}{8}-\frac{P}{2}\right)\right) = 0$$

$$y = \frac{1}{EI}\left(\frac{\frac{P}{2}L^3}{48} + \frac{\frac{L^2}{6}\left(\frac{P}{8}-\frac{P}{2}\right)L}{2}\right) = -\frac{PL^3}{48EI}$$